Why CCSS-M Grades K-3 is Developmentally Appropriate and Internationally Competitive
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Part 1: How did the CCSS-M development process draw on teachers and experts in early childhood math education?

Here are some important facts to know provided by Jason Zimba, a lead CCSS-M author:

Together, the working group and feedback group for the Common Core State Standards included:

- State directors of state P–16 and P–20 programs, responsible for pre-K and early elementary education in their states.
- Current and former public school elementary teachers.
- National experts on early childhood math education, including Karen Fuson and Douglas Clements.
- Input from experienced K–2 teachers was solicited and used. For example, the connections between numbers and the quantities they name were improved thanks to the input of K–2 teachers who were part of a larger group convened by the American Federation of Teachers for the purpose of providing feedback on an early draft.
- Experts in early childhood education from other organizations also gave feedback on the public draft in September 2009 and March 2010 (when more than 10,000 individuals contributed feedback). This led to concrete changes in the final version. For example, the concept of a tens unit was moved from kindergarten to grade 1 in response to feedback from early childhood educators.
- Research on early learning strongly informed the development of the standards. One important source was the National Research Council's 2009 report, *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, cited in the bibliography of the Standards. Many other titles relating to early childhood math instruction can be found on page 91 of the CCSS-M document. Titles include works such as *Engaging young children in mathematics: Standards for early childhood mathematics education* and *Children's Mathematics: Cognitively Guided Instruction.*

These sample observations understate the involvement of early childhood educators in the development process, because public school teachers were also asked by state education agencies to contribute to reviews of preliminary drafts.

Issues concerning "Developmentally Appropriate and Common Core at Elementary Grades"

Point 1: *Standards typical state a clear target in the first sentence that describes the expectation.*

After that, the Standards include *suggestions* for research-backed strategies for learning to ensure that the students' learning is made as conceptually rich and efficient as possible. Math is a language of connections.

Here is a typical first grade standard: "Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8+6 = 8+2+4 = 10+4 = 14); decomposing a number leading to a ten (e.g., 13-4 = 13-3-1 = 9)…and creating equivalent but easier or known sums (e.g., adding 6+7 by creating the known equivalent 6+6+1 = 12+1 = 13)."

The cognitive strategies have widespread documentation (for example, see *Adding It Up*, NRC*).* This work, often by cognitive scientists and learning scientists, shows how children solve these problems most insightfully and successfully. They are evidence that mathematical thinking involves a rich set of connections among ideas.

Point 2: *The Standards are not meant to be read to children.*

Would you expect a child to understand the technical knowledge provided to a doctor? The standards treat teachers as professionals and fully capable of learning of understanding the terminology. For example, the language of composing and decomposing can be unfamiliar to teachers but it is used in three ways consistently in the standards: to compose and decompose into place value units (13 is one ten and three ones), to compose and decompose geometric figures and, later in high school, students to compose and decompose functions.

Point 3: *Standards identify research-based strategies for student success.*

Many developmental psychologists support the Common Core as developmentally appropriate. The **National Association for the Education of Young Children** in conjunction with the **National Association of Early Childhood Specialists in States** issued a joint statement publicly expressing their support for the Standards. Some developmental psychologists disagree. This may be in part because some developmental psychologists know mathematics minimally and their work focuses on documenting what children show them in limited settings. Working in labs, they lack a broad set of opportunities to observe children in classrooms. Others are not aware that the strict stage theory in Piaget has evolved in favor of more continuous development. Author, researcher and Professor of Psychology at the University of Virginia Daniel Willinghamm[1] argues that data from the last twenty years shows that cognitive development is continuous rather than occurring in discrete stages, implying that students are capable of comprehending complex concepts and challenging material even in the youngest grades. Most important are his views of building schemes and operations that require that operations derive from actions, involve finding an invariant in a variety of context, exhibits structure and can be reversed. As Vygotsky argued in his concept of a "zone of proximal development", instruction needs to lead development. Many researchers having both training in developmental and cognitive psychology and math knowledge have demonstrated young children are capable of learning the ideas in the Standards IF they are given opportunities to use models and manipulative, to develop and explore their own strategies, provided opportunities to learn to write and describe those ideas in numeric form and are provided opportunities to practice those skills both mentally and in written form.

Point 4: *The Standards have a very solid approach to learning.*

These strategies have been reported in NMAP and "Adding It Up" and used for years by the strongest teachers. The Standards help clarify them for all teachers. Children build ideas in relation to using manipulatives and different models (for example two models of subtraction are $a-b=c$ means take a quantity "a" and remove "b" items to end up with "c" items. Or $a-b=c$ refer to two points on a number line a and b, and c is their "difference," or the distance between them counted up from b to a. Given the models and manipulatives, students learn to recognize an operation in a context. They work with mental math, using strategies such as counting on (to solve 6+3, you start at 6 and count up 7,8,9) or using known facts (6+7= 6+6 +1) Then they learn to write it and the related rules for writing it as a number sentence. Then they learn to "reverse it" or work with the inverse operation. (If $a-b=c$ then $c+b=a$). The

[1] http://www.danielwillingham.com/daniel-willingham-science-and-education-blog/what-is-developmentally-appropriate

standards also expect students to develop fluency in every basic operation (see Confrey 2015, response to Milgram and Wurman).

Point 5: *The key concepts are designed to build in complexity across the grades at an appropriate pace of increasing challenge.*

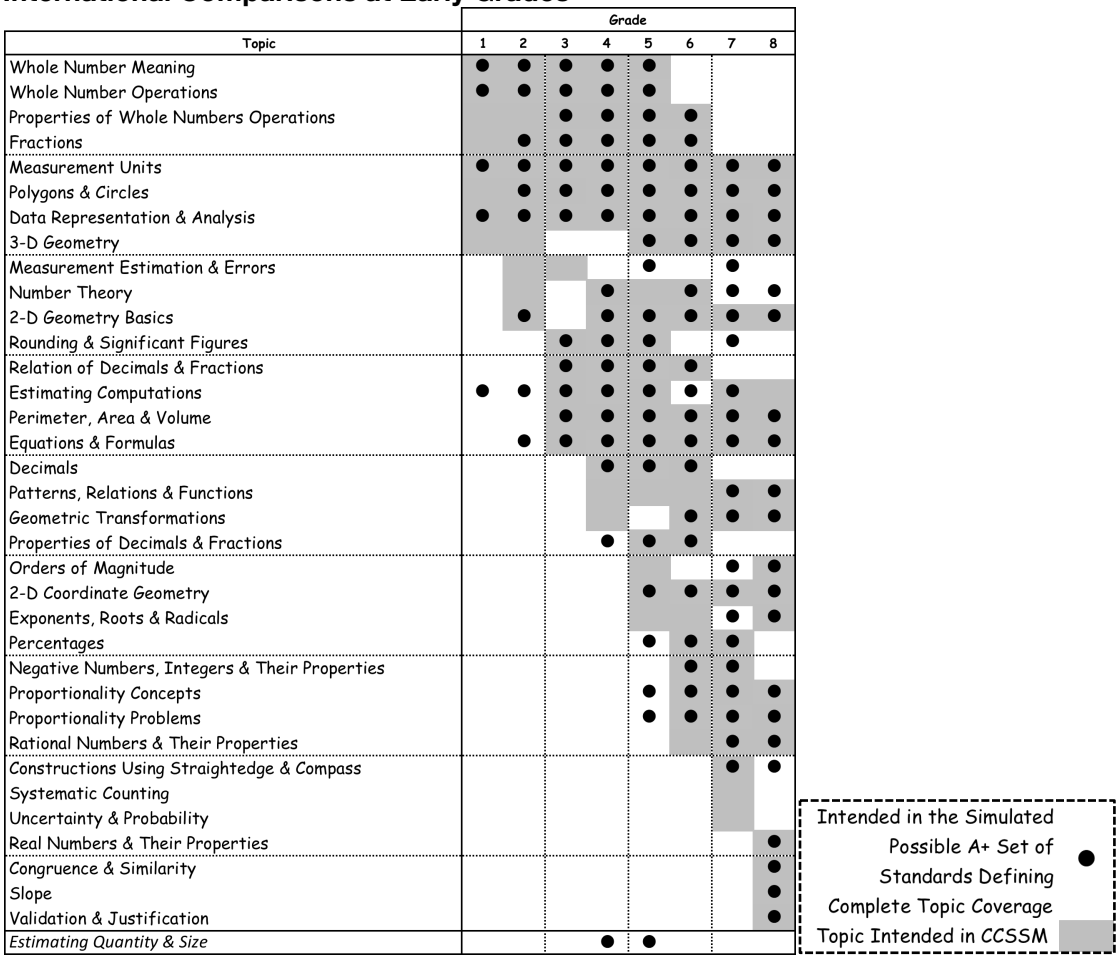
The Progressions/Learning Trajectories are at the heart of the standards. Attached is a spreadsheet for the K-6 elementary standards displayed horizontally into a sequence by strand such as by place value or by applied problems etc. Confrey and Maloney (2012) built these to display the standards by grade level in a way that also allows one to look across to see how the standards build across the grades. They are available for middle and high school too. These are sold by "Amplify."[2] When analyzed into these sequences, it becomes readily apparent that the standards increase in difficulty at an appropriate pace across the grades.

Point 6: *The Standards are consistent with high performing countries.*

To examine this point, a chart from William Schmidt, a member of the TIMSS (Third International Math and Science Study) and a member of the National Validation Committee. It compares the CCSS-M with the standards of the top performing international countries. Below that are four charts adapted from ones in the AIR (2009) Report "Informing Grades 1-6 mathematics standards development: What can be learned from high performing Hong Kong, Korea, and Singapore" (Mullis et al. 2012). AIR took the three countries of Hong Kong, Singapore and Korea and integrated their standards to create a composite set of standards. These are reproduced for the early elementary grades of K-3 for "whole number," "adding and subtracting," "length," and "shapes." These topics are the primary ones in the Standards. Added below each grade of the composite standards is written the corresponding CCSS-M standard. Note the other countries do not have a Kindergarten, but since CCSS-M does, this is also shown. Note that in Kindergarten, American students start earlier, but they are on par with the other countries starting in their comparable grades. Thus, in fact, American children have a more gentle ramp to learning the content, due to the Kindergarten opportunity. If they are consistent with other countries and people want to claim they are not developmentally appropriate, then those people are asserting our children cannot learn what other children learn.

[2] http://store.amplify.com/learning-trajectory-posters-c5.aspx

Issues Concerning the International Comparisons at Early Grades



In the figure above, the topics on the left are shown with a black dot, where they occur in the top performing countries. Location of the topics in CCSS-M are shaded in grey. (Schmidt and Houang (2012) Educational Researcher p. 296.)

Below are descriptions of the composite standards of Hong Kong, Singapore and Korea and below them is the same description for Common Core at that same grade.

General Description of the Composite Standards

The major topics in the numbers strand for all three countries follow a similar pattern, across grades, dictated by the logic of mathematics learning. The standards first introduce whole numbers and place value, starting with numbers to 100. The presentation of whole numbers continues over the grades, with students systematically exposed to a larger number of digits. The standards simultaneously introduce arithmetic operations of addition and subtraction followed by multiplication and division, with the number of digits of arithmetic problems expanding in tandem with students' exposure to whole numbers and place value. Fractions, decimals, and percentages follow once students have a firm grasp of basic whole number concepts and operations.

This same array of topics and order are used in CCSS-M.

Understanding and Reading Whole numbers in the Composite Standards.

Table 6. Composite Standards for Hong Kong, Singapore and South Korea, with the Addition of the CCSS-M. Composite Standards: Numbers—Whole Numbers for Hong Kong, Singapore, South Korea (AIR, p. 8)

K	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5
HK, S, SK	**HK, S, SK**	**HK, S, SK**	**HK, S, SK**	**HK, S, SK**	**HK, S, SK**
	Whole numbers to 100: • Count to tell the number of objects in a given set • Count forward and backward • Compare the number of objects in two or more sets • Use ordinal numbers (first, second, up to tenth) and symbols (1st, 2nd, 3rd, etc.)	Whole numbers to 1,000: • Count in tens and hundreds • Use number notation and place values (hundreds, tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers	Whole numbers to 10,000: • Use number notation and place values (thousands, hundreds, tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers • Understand odd and even numbers	Whole numbers to 100,000: • Use number notation and place values (ten thousands, thousands, hundreds, tens, ones) • Read and write numbers in numerals and in words • Compare and order numbers • Round numbers to	Develop an understanding of large numbers: • Develop the concept of approximation • Estimate the number of a large quantity of objects • Round large numbers in thousands, ten thousands, hundred thousands, millions, ten millions,

	• Use number notation and place values (tens, ones) • Read and write numbers in numerals and in words Compare and order numbers			the nearest 10 or 100	hundred millions
CCSS-M	**CCSS-M**	**CCSS-M**	**CCSS-M**	**CCSS-M**	**CCSS-M**
Whole numbers: • Count to 100 by ones and tens • Count forward from a given number • Write numbers 0-20 • Represent number of objects with numerals 0-20 • Count to answer "how many?" of a group of objects • Connect counting to cardinality • Compare the numbers of objects in two groups of objects, up to 10 • Compare written numerals between 1 and 10	Whole numbers: • Count to 120 • Read and write numerals 0-120 • Represent a number of objects with numeral	Whole numbers: • Count within 1000 • Skip-count by 5s, 10s, and 100s • Read and write numbers to 1000 using base-ten numerals, number-names, and expanded form			

Addition and Subtraction of whole numbers grades 1-5

Table 7. Composite Standards for Hong Kong, Singapore and South Korea with the addition of CCSS-M: Numbers—Addition and Subtraction (AIR, p. 9)

K	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5
HK, S, SK	**HK, S, SK**	**HK, S, SK**	**HK, S, SK**	**HK, S, SK**	**HK, S, SK**
	Addition and subtraction: • Understand situations for, and the meaning of, addition and subtraction • Use the addition symbol (+) or the subtraction symbol (–) • Compare two numbers within 20 to tell how much one number is greater (or smaller) than the other • Recognize the relationship between addition and subtraction • Build the addition bonds up to 9 + 9 • Solve1-step word problems involving addition and subtraction within 20 • Add more than two 1-digit numbers • Add and subtract within 100 without regrouping involving • – a 2-digit number	Addition and subtraction of numbers up to three digits: • Solve up to 2-step word problems involving addition and subtraction • Use mental calculation for addition and subtraction involving • – a 3-digit number and ones • – a 3-digit number and tens • – a 3-digit number and hundreds	Addition and subtraction of numbers up to four digits: • Use the terms “sum” and “difference” • Solve up to 2-step word problems involving addition and subtraction		

	and ones • – a 2-digit number and tens • – two 2-digit numbers • Use mental calculation for addition and subtraction • – within 20 • – involving a 2-digit number and ones without renaming • – involving a 2-digit number and tens				
CCSS-M	**CCSS-M**	**CCSS-M**	**CCSS-M**	**CCSS-M**	**CCSS-M**
Whole numbers: • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. • Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (*e.g.*, claps), acting out situations, verbal explanations, expressions, or equations. • Decompose numbers less than or equal to 10 into pairs in more than	Whole numbers: • Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). • Apply properties of operations as strategies to add and subtract. • Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten, decomposing a number leading to a ten, using the relationship between addition and	Whole numbers: • Understand and use place value understanding and properties of operations to add and subtract. • Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • Add up to four two-digit numbers using strategies based on place value and	Whole numbers: • Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.		

one way, *e.g.*, by using objects or drawings, and record each decomposition by a drawing or equation (*e.g.*, 5 = 2 + 3 and 5 = 4 + 1). • For any number from 1 to 9, find the number that makes 10 when added to the given number, *e.g.*, by using objects or drawings, and record the answer with a drawing or equation.	subtraction, and creating equivalent but easier or known sums • Understand subtraction as an unknown-addend problem. • Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/ or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	properties of operations. • Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.) • Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.			

Length within Measurement

Table 4. Composite Standards for Hong Kong, Singapore and South Korea, with the Addition of CCSS-M. Composite Length Within Measurement (AIR, p.6)

K	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5
HK, S, SK	**HK, S, SK**	**HK, S, SK**	**HK, S, SK**	**HK, S, SK**	**HK, S, SK**
	Concept of length: • Understand concepts of length and distance • Understand long, longer, longest, short, shorter, shortest • Understand centimeter Tools/measuring length: • Measure and compare lengths of objects and distance with centimeters • Estimate lengths and distances • Measure length with appropriate tools	Concept of length: • Understand that a meter is greater than a centimeter Tools/measuring length: • Measure and compare length and distance in meters and centimeters • Estimate lengths and distances • Measure length with appropriate tools	Concept of length: • Understand that a kilometer is greater than a meter and that a millimeter is smaller than a centimeter • Convert compound units to a smaller or a larger unit Tools/measuring length: • Measure and compare length and distance in kilometers and millimeters • Estimate lengths and distances • Measure length with appropriate tools		• Convert from a smaller unit to a larger unit and vice versa in decimal form
CCSS-M	**CCSS-M**	**CCSS-M**	**CCSS-M**	**CCSS-M**	**CCSS-M**
• Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. • Directly compare two objects with a	• Order three objects by length; compare the lengths of two objects indirectly by using a third object. • Express the length of an object as a whole number of length units, by laying multiple	• Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. • Measure the length of an object twice, using length units of different		• Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a	• Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world

measurable attribute in common, to see which object has “more of”/”less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.	copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.	lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. • Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. • Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram • Estimate lengths using units of inches, feet, centimeters, and meters.		larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	problems.

Two-Dimensional Shapes in Geometry

Table 19 Table 4 Composite Standards for Hong Kong, Singapore and South Korea, with the Addition of CCSS-M. Composite Standards: Geometry—Two-Dimensional Shapes (AIR, p. 26)

K	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5
HK, S, SK	HK, S, SK	HK, S, SK	HK, S, SK	HK, S, SK	HK, S, SK
	Four basic shapes: rectangle, square, circle, triangle • Identify and name the four basic shapes from 2-dimensional and 3-dimensional objects, describing and classifying shapes Patterns: • Make or complete patterns with 2-dimensional cut-outs according to one or two of the following attributes: – shape – size – color	• Identify the basic shapes that make up a given figure • Form different 2-dimensional figures with cut-outs of – rectangle – square – triangle – semicircle – quarter circle • Copy figures on dot grid or square grid Patterns: • Make or complete patterns with 2-dimensional cut-outs according to one or two of the following attributes: – shape – size – orientation – color		Rectangle and square: • Understand the properties of a rectangle and a square • Find unknown angles Symmetry: • Identify symmetric figures • Determine whether a straight line is a line of symmetry of a symmetric figure and complete a symmetric figure with respect to a given horizontal or vertical line of symmetry	Triangle: • Identify and name the following types of triangles: – isosceles triangle – equilateral triangle – right angle triangle • Use the property that the angle sum of a triangle is 180o to find unknown angles • Draw a triangle from given dimensions using ruler, protractor, and set squares Parallelogram, rhombus, and trapezoid: • Identify and name parallelogram, rhombus, and trapezoid • Understand the properties of parallelogram, rhombus, and trapezoid • Find unknown angles • Draw a square,

					rectangle, parallelogram, rhombus, or trapezoid from given dimensions using ruler, protractor, and set squares
CCSS-M	**CCSS-M**	**CCSS-M**	**CCSS-M**	**CCSS-M**	**CCSS-M**
• Describe objects in the environment using names of shapes (squares, circles, triangles, rectangles, hexagons, • cubes, cones, cylinders, and spheres), and describe the relative positions of these objects using terms such as above, below, • beside, in front of, behind, and next to. • Correctly name shapes regardless of their orientations or overall size. • Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?" • Model shapes in the world by building	• Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes • Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal	• Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	• Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories	• Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. • Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. • Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts.	• Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles • Classify two-dimensional figures in a hierarchy based on properties.

shapes from components (e.g., sticks and clay balls) and drawing shapes • Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”). • Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/”corners”) and other attributes (e.g., having sides of equal length)	names such as “right rectangular prism.”)			Identify line-symmetric figures and draw lines of symmetry	

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